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Original article

Оригинальная статья

## STRESS-STRAIN STATE OF CERAMIC SHELL MOLD DURING FORMATION OF SPHERICAL STEEL CASTING IN IT. PART 2

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**Abstract.** The paper presents the results of numerical calculations of the solution to the problem of modeling the process of possible cracking in a spherical shell mold when pouring liquid steel into it and cooling the solidifying casting. The numerical scheme of the axisymmetric problem and the algorithm for its solution were given in Part 1. The crack resistance is estimated by magnitude of the normal stresses in the ceramic shell during its co-cooling with a solidifying casting. The results detailed analysis considered: fields of displacement, stresses, and temperatures both on spherical surface and in growing crust of solidified metal. The solution took into account the change in the shear modulus of the mold material from temperature, and an assessment of this refinement was given. The problem was solved in two ways. The first – with a constant shift modulus of the shell mold; the second – with its temperature-dependent shift modulus. There is a significant difference between these variants in terms of magnitude of the normal stresses arising in the shell mold. The authors analyzed resistance of the shell mold spherical geometry to external influences from its support filler and filling funnel. The problem of determining the contact and free surfaces at the boundary of the shell mold and support filler was solved. The results are presented graphically in the form of diagrams of stresses and temperatures over the studied area in its different sections and time intervals for cooling of the growing metal crust. The role of compressive normal stresses  $\sigma_{22}$ ,  $\sigma_{33}$  on the surface of contact of the shell mold with liquid metal at the initial moment of cooling on probability of cracking in a spherical mold is shown. The level of strain-stress state in a spherical shell mold when cooling a steel casting in it is significantly determined by dependence of shift modulus of the shell mold on temperature.

**Keywords:** investment casting, shell mold, stressed state, modeling, cracking

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## НАПРЯЖЕННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ КЕРАМИЧЕСКОЙ ОБОЛОЧКОВОЙ ФОРМЫ ПРИ ФОРМИРОВАНИИ В НЕЙ СТАЛЬНОЙ ШАРООБРАЗНОЙ ОТЛИВКИ. ЧАСТЬ 2

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**Аннотация.** В статье приведены результаты численного решения задачи по моделированию процесса возможного трещинообразования в оболочковой форме (ОФ) шарообразной конфигурации при заливке в нее жидкой стали и охлаждении затвердевающей отливки. Численная схема осесимметричной задачи и алгоритм решения были приведены в части 1. Трещиностойкость оценивается

по величине нормальных напряжений в керамической ОФ в процессе ее совместного охлаждения с затвердевающей отливкой. При детальном анализе результатов были учтены поля перемещений, напряжений, температур как в сферической ОФ, так и в нарастающей корочке затвердевшего металла. При решении учитывалось изменение модуля сдвига материала формы от температуры, и была дана оценка этого уточнения. Задачу решали двумя способами. Первый – с постоянным модулем сдвига ОФ; второй – с модулем сдвига ОФ, зависящим от температуры. Между этими вариантами есть существенная разница в величине нормальных напряжений, возникающих в ОФ. Авторы проанализировали стойкости ОФ сферической геометрии от внешних воздействий со стороны опорного наполнителя (ОН) оболочковой формы и заливочной воронки. Была решена задача по определению контактной и свободной поверхностей на границе ОФ и ОН. Результаты решения задачи представлены графически в виде эпюра напряжений, температур по исследуемой области в разных ее сечениях и временных интервалах охлаждения ОФ и нарастающей корочки металла. Показана роль сжимающих нормальных напряжений  $\sigma_{22}$ ,  $\sigma_{33}$  на поверхности соприкосновения ОФ с жидким металлом в начальный момент охлаждения на вероятность трещинообразования в сферической форме. Уровень напряженно-деформированного состояния в сферической ОФ при охлаждении в ней стальной отливки существенно определяется зависимостью модуля сдвига ОФ от температуры.

**Ключевые слова:** литье по выплавляемым моделям, оболочковая форма, напряженное состояние, моделирование, трещинообразование

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## INTRODUCTION

Previous studies on the influence of the stress-strain state (SSS) on the crack resistance of shell molds (SM) during the pouring of liquid metal and subsequent cooling with the solidifying casting were conducted using shell molds for investment casting (IC) in the form of risers with both cylindrical and spherical (sump) shapes. Numerous theoretical and experimental investigations have been carried out to identify the features of the SSS in ceramic shell molds and the resulting castings in investment casting. These studies examined various factors, including the materials used in the investment models [1; 2], the shape and geometry of the SM [3; 4], the mold wall thickness [5; 6], the mold material [7; 8], the geometry of the castings [9 – 11], methods for testing mold strength, and more [12; 13].

Mathematical modeling of these processes has also been presented in other works, covering modeling methods [14], research [15 – 17], numerical modeling [18 – 20], specialized mathematical models [21 – 23], and software tools [24; 25].

The production of spherical and globular IC castings, and consequently the resistance of the SM to cracking during the formation of such castings, is of both scientific and practical interest. The materials in this study are focused on addressing this issue.

In [26], the general approach to constructing a mathematical model for determining the SSS and temperature in an SM during the cooling of a spherical casting was presented, along with the numerical scheme and algorithm for solving the problem using developed software packages [27; 28]. This study presents the results of theoretical and numerical research aimed at solving the outlined problem.

## MAIN BODY

In [26], the general problem of cooling a spherical casting in a shell mold was described.

Fig. 1 illustrates the computational scheme of the process under study, considering axial symmetry (where the angle  $\alpha$  defines the size of the gating window, and the angle  $\varphi$  defines the extent of the SM's coverage by the supporting filler (SF)).

### Initial date

Geometric parameters:  $S = 5$  mm,  $R_1 = 20$  mm.

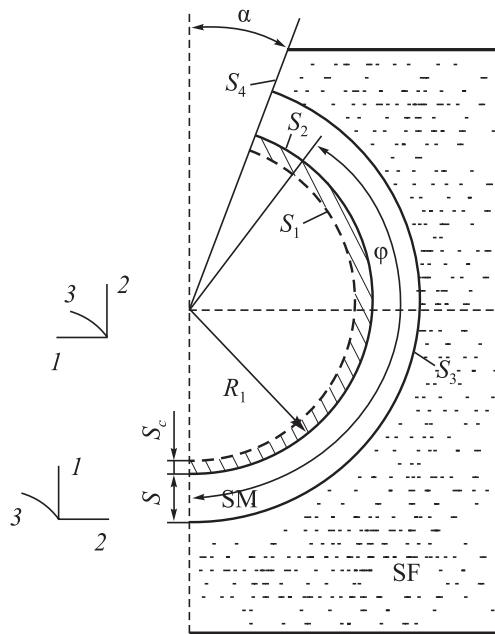
Time intervals  $\Delta t_n$ : 0.1; 1.0; 2.0; 4.0; 5.0; 5.0; 5.0; 10.0; 10.0; 1.0; 2.0; 5.0; 1.0; 1.0; 3.0; 3.0; 5.0; 10.0; 10.0 s; the friction parameter on surface  $S_3$  (Fig. 1) is  $\psi = 0.001$ .

Domain partitioning:  $N_1 \times N_2 = 10 \times 30$ .

Accepted physical parameters of the cast steel at a temperature of  $\theta > 1000$  °C ( $\theta_m^* = 1500$  °C):  $G = 1000$  kg/mm<sup>2</sup> (shear modulus);  $\alpha = 12 \cdot 10^{-6}$  °C<sup>-1</sup> (coefficient of linear expansion);  $\lambda = 0.0298$  W/(mm·°C) (thermal conductivity coefficient);  $L = 270 \cdot 10^3$  J/kg (latent heat of fusion);  $C = 444$  J/(kg·°C) (specific heat capacity);  $\gamma = 7.80 \cdot 10^{-6}$  kg/mm<sup>3</sup> (density);  $\theta_s = 1450$  °C (solidification temperature).

Physical properties of the ceramic mold:  $G_m = 2910$  kg/mm<sup>2</sup>;  $\alpha = 0.51 \cdot 10^{-6}$  °C<sup>-1</sup>;  $\lambda = 0.000812$  W/(mm·°C);  $C = 840$  J/(kg·°C);  $\gamma = 2.0 \cdot 10^{-6}$  kg/mm<sup>3</sup>.

Some theoretical studies [29] have shown that during the cooling of steel in an SM with  $\alpha$  angles of 10 and 30° and  $\varphi = (180^\circ - \alpha)$ , significant compressive stresses  $\sigma_{22}$  and  $\sigma_{33}$  can occur, potentially exceeding the compressive strength of the ceramic material. When  $\alpha = 30^\circ$ , the stresses  $\sigma_{ii}$ ,  $i = 2, 3$  are slightly lower in absolute value



**Fig. 1.** Calculation scheme of a spherical ceramic shell mold molded in support filler and filled with liquid metal, taking into account axial symmetry:

LM – liquid metal; TM – solid metal; SM – shell mold;

SF – support filler;  $S_1$  – inner contact surface of liquid and solidified metal;  $S_2$  – inner contact surface of solidified metal and shell mold;

$S_3$  – outer surface of shell mold;  $S_4$  – free surface of the end face of casting cup;  $R_1$  – radius of spherical casting;  $S$  – thickness of shell mold;  $S_T$  – thickness of solidified metal crust;  $\alpha$  – slope angle of funnel;  $\varphi$  – angle of enclosing surface of shell mold with a support filler

**Рис. 1.** Расчетная схема шарообразной ОФ, заформованной в опорный наполнитель и залитой жидким металлом, с учетом осевой симметрии:

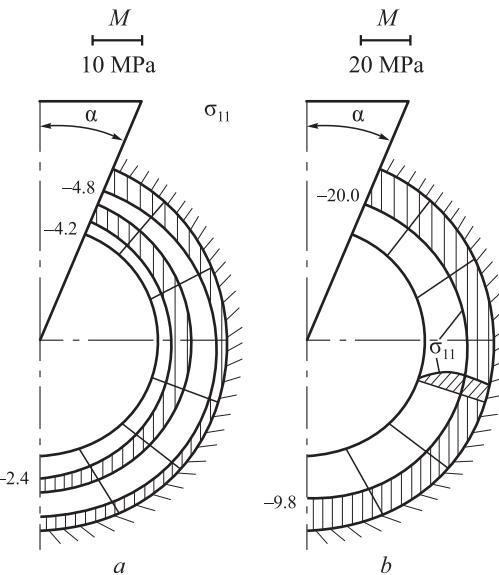
LM – жидкий металл; TM – твердый металл;

SF – оболочковая форма; TM – опорный наполнитель;  $S_1$  – внутренняя поверхность контакта жидкого и затвердевшего металла;  $S_2$  – внутренняя поверхность контакта затвердевшего металла и оболочковой формы;  $S_3$  – внешняя поверхность оболочковой формы;  $S_4$  – свободная поверхность торца литниковой чаши ОФ;  $R_1$  – радиус шарообразной отливки;  $S$  – толщина оболочковой формы;  $S_T$  – толщина корочки затвердевшего металла;  $\alpha$  – угол наклона литниковой воронки;  $\varphi$  – угол охвата поверхности оболочковой формы опорным наполнителем

than at  $\alpha = 10^\circ$ . Fig. 2 shows the distribution of normal stresses  $\sigma_{ii}$  in the SM at  $\alpha = 10^\circ$ : (a) after 7.1 s of cooling, (b) after 51.1 s.

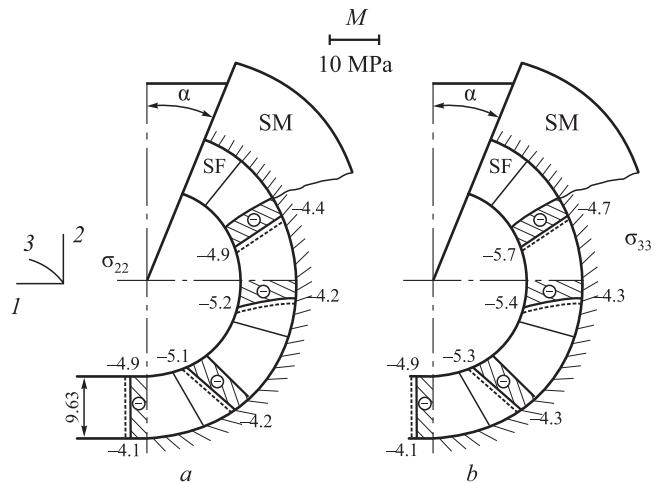
Fig. 3 illustrates the distribution of stresses  $\sigma_{22}$  and  $\sigma_{33}$  in the solidified metal shell after 60.1 s of cooling; solid lines represent the distributions at  $\alpha = 30^\circ$ , while dashed lines represent those at  $\alpha = 10^\circ$ .

The values of  $\sigma_{22}$  and  $\sigma_{33}$  on the surface adjacent to the liquid metal are greater (in magnitude) than on the surface of the SM, which is explained by the constant shear modulus  $G_m$  of the forming solid metal ( $1000 \text{ kg/mm}^2$ ), independent of temperature. As in the SM, the stresses  $\sigma_{22}$  and  $\sigma_{33}$  are compressive, and at  $\alpha = 10^\circ$ , they are higher than at  $30^\circ$ .



**Fig. 2.** Диаграммы нормальных напряжений  $\sigma_{11}$  в ОФ при  $\alpha = 10^\circ$  при времени охлаждения отливки 7,1 с (а) и 51,1 с (б)

**Рис. 2.** Эпюры нормальных напряжений  $\sigma_{11}$  в ОФ при  $\alpha = 10^\circ$  при времени охлаждения отливки 7,1 с (а) и 51,1 с (б)



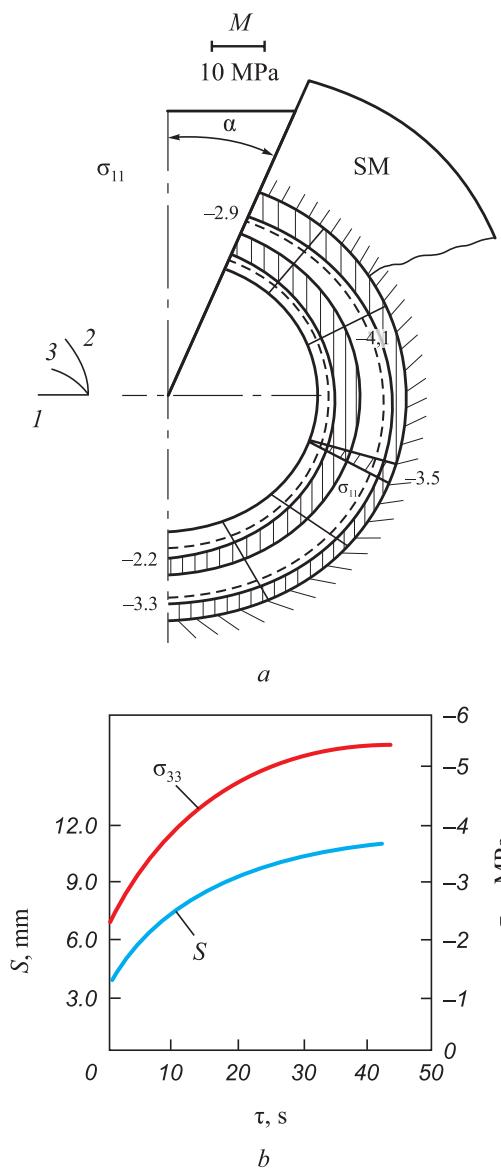
**Fig. 3.** Диаграммы нормальных напряжений  $\sigma_{22}$  (а) и  $\sigma_{33}$  (б) в образующей корочке металла при  $\alpha = 30^\circ$  (сплошные линии) и  $\alpha = 10^\circ$  (штриховые линии) и времени охлаждения отливки 60,1 с

**Рис. 3.** Эпюры нормальных напряжений  $\sigma_{22}$  (а) и  $\sigma_{33}$  (б) в образующей корочке металла при  $\alpha = 30^\circ$  (сплошные линии) и  $\alpha = 10^\circ$  (штриховые линии) и времени охлаждения отливки 60,1 с

Fig. 4, a shows the distribution of stresses  $\sigma_{11}$  in the solidified metal skin after 60.1 s. The stresses  $\sigma_{11}$  are compressive throughout the cross-section: solid lines indicate  $\sigma_{11}$  at  $\alpha = 30^\circ$ , while dashed lines show them at  $\alpha = 10^\circ$ .

Fig. 4, b shows the growth curves of the skin thickness ( $S$ ) and the stress  $\sigma_{33}$  over time. As the angle  $\alpha$  increases, the normal stresses decrease slightly (in magnitude).

Further increasing  $\alpha$  is unnecessary, as the metal skin continues to grow with only minor changes in normal stresses.



**Fig. 4.** Диаграммы нормальных напряжений  $\sigma_{11}$  (а) в кристаллизованной корочке металла после литья и охлаждения на 60,1 с и кривые роста величины корочки металла ( $S$ ) и напряжений  $\sigma_{33}$  (б) со временем охлаждения

**Рис. 4.** Эпюры нормальных напряжений  $\sigma_{11}$  (а)

в закристаллизованной корочке металла через время охлаждения отливки 60,1 с и кривые роста величины корочки металла ( $S$ ) и напряжений  $\sigma_{33}$  (б) со временем охлаждения

The results were obtained assuming a constant (average) shear modulus for the SM ( $G_m$ ).

As indicated in [29], the temperature in the SM adjacent to the solidifying metal is very high (around 1300 °C). At this temperature, the ceramic is practically in a softened state, meaning the shear modulus in this area will be much lower than the average value of  $G_m$  for the shell mold. We will use experimental data (Fig. 5) obtained in [30] from testing ceramic samples made from a binder material ( $\text{SiO}_2 + \text{MgPO}_4$ ).

Approximating the results presented in Fig. 5, we obtain:

$$G_m = 6412 - 6.37\theta, \text{ kg/mm}^2 \text{ for } 300^\circ\text{C} < \theta < 1000^\circ\text{C};$$

$$G_m = 40 \text{ kg/mm}^2 \text{ for } \theta \geq 1000^\circ\text{C};$$

$$G_m = 4500 \text{ kg/mm}^2 \text{ for } \theta < 300^\circ\text{C}.$$

The results of the solution with a temperature-dependent shear modulus of the SM are shown in Fig. 6 at  $\alpha = 10^\circ$  as distributions of  $\sigma_{ii}$ ,  $i = 1, 2, 3$  for  $\tau = 1.12$  s (solid lines);  $\tau = 7.12$  s (dashed lines).

During thermal shock and cooling at  $\tau = 1.12$  s, the stresses  $\sigma_{22}$  and  $\sigma_{33}$  have the highest values (in absolute terms) in the SM's contact zone with the metal, but they change sharply with cooling time. All normal stresses are compressive. The stresses  $\sigma_{33}$  in the forming skin ( $S = 6$  mm) after  $\tau = 7.12$  s are shown in Fig. 7.

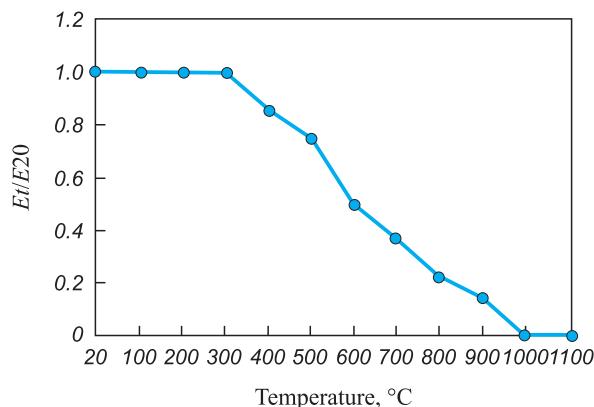
With further cooling, the stresses  $\sigma_{ii}$  decrease.

The most critical period for failure is the initial cooling phase ( $0 < \tau < 8$  s) (Fig. 6).

Figs. 8 and 9 show the calculation results for  $\alpha = 30^\circ$ ,  $\tau = 1.12$  s (solid lines);  $\tau = 7.12$  s (dashed lines).

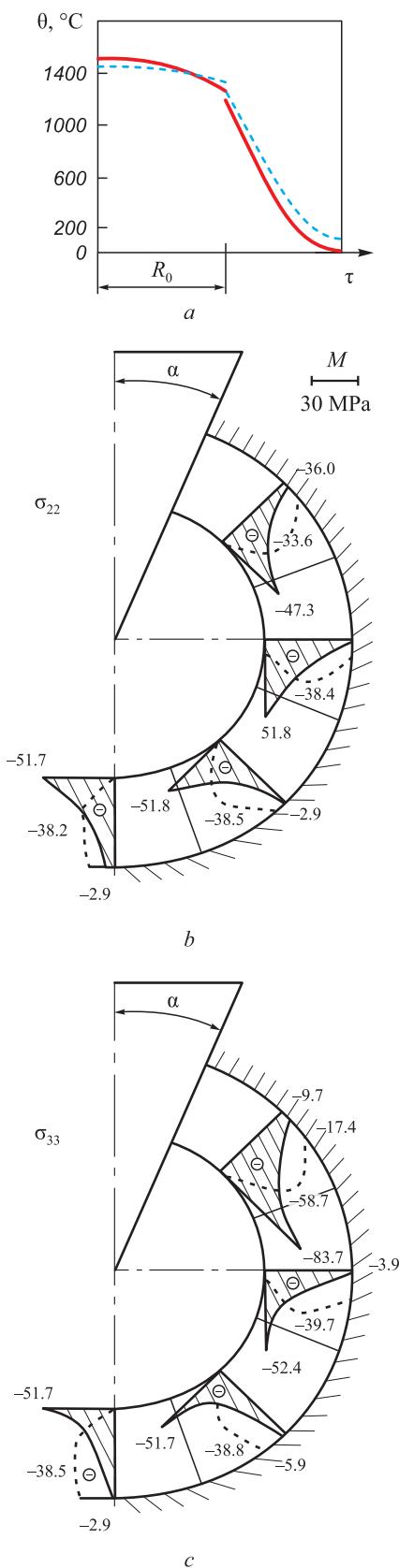
As seen, the pattern is approximately the same, but the values of  $\sigma_{ii}$ ,  $i = 1, 2, 3$  are slightly lower than at

$\text{SiO}_2 + \text{MgPO}_4$				
$T$	$E$	$Et/E20$	$G, \text{ MPa}$	Vol. expansion coefficient
20	11250000000	1.0000	45,000	4.4444 E-06
100	11250000000	1.0000	45,000	4.4444 E-06
200	11250000000	1.0000	45,000	4.4444 E-06
300	11250000000	1.0000	45,000	4.4444 E-06
400	9642857143	0.8571	38,571	5.18519 E-06
500	8437500000	0.7500	33,750	5.92593 E-06
600	5625000000	0.5000	22,500	8.88889 E-06
700	4218750000	0.3750	16,875	1.18519 E-05
800	2596153846	0.2308	10,385	1.92593 E-05
900	1687500000	0.1500	6750	2.96296 E-05
1000	96428571	0.0086	386	5.18519 E-04
1100	61363636	0.0055	245	8.14815 E-04



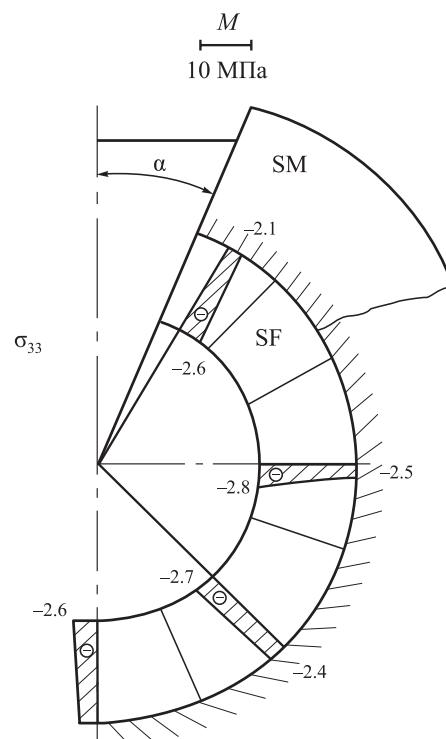
**Fig. 5.** Экспериментальные данные при испытании керамических образцов, выполненных из связующего материала ( $\text{SiO}_2 + \text{MgPO}_4$ )

**Рис. 5.** Экспериментальные данные при испытании керамических образцов, выполненных из связующего материала ( $\text{SiO}_2 + \text{MgPO}_4$ )



**Fig. 6.** Temperatures field (a), diagrams of normal stresses  $\sigma_{22}$  (b) and  $\sigma_{33}$  (c) in ceramic mold at time of casting cooling 1.12 s (—) and 7.12 s (---)

**Рис. 6.** Поле температур (а), эпюры нормальных напряжений  $\sigma_{22}$  (б) и  $\sigma_{33}$  (в) в ОФ при  $\alpha = 10^\circ$  и времени охлаждения отливки 1,12 с (—) и 7,12 с (---)



**Fig. 7.** Diagrams of normal stresses  $\sigma_{33}$  in resulting metal crust ( $S = 6 \text{ mm}$ ) at time of casting cooling 7.12 s

**Рис. 7.** Эпюры нормальных напряжений  $\sigma_{33}$  в образующейся металлической корочке ( $S = 6 \text{ мм}$ ) при времени охлаждения отливки 7,12 с

$\alpha = 10^\circ$ , and small tensile stresses  $\sigma_{22}$  and  $\sigma_{33}$  have even appeared in the SM at the interface with the SF (Fig. 8). In the metal skin ( $S = 6 \text{ mm}$ ) after  $\tau = 7.12$  s, the stresses  $\sigma_{33}$  are shown (Fig. 9), which are also lower than at  $\alpha = 10^\circ$  (Fig. 3, b).

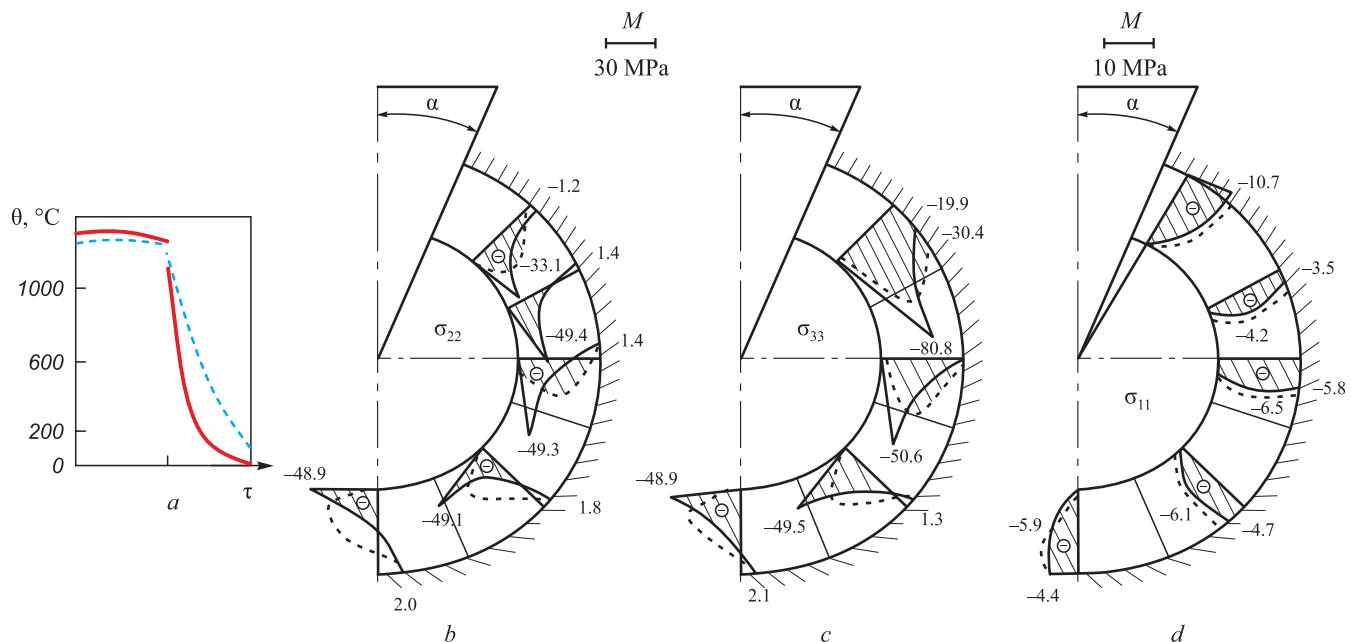
Further cooling shows that normal stresses decrease, and the distributions of  $\sigma_{ii}$  for  $\tau = 32.12$  and  $52.12$  s are close to each other.

Regarding the effect of the cylindrical SM material on its durability, these results were presented in previous works by the authors, where in the case under consideration, the most critical factor for potential crack formation in the SM is the tensile normal stresses  $\sigma_{22}$  in the outer layer of the shell, which is in contact with the supporting filler.

Considering the temperature dependence of the shear modulus in the SM significantly affects the stress-strain state during the cooling of the steel casting within it. Under the given external conditions for the metal cooling process in a spherical SM, its durability at the initial moment of pouring is questionable.

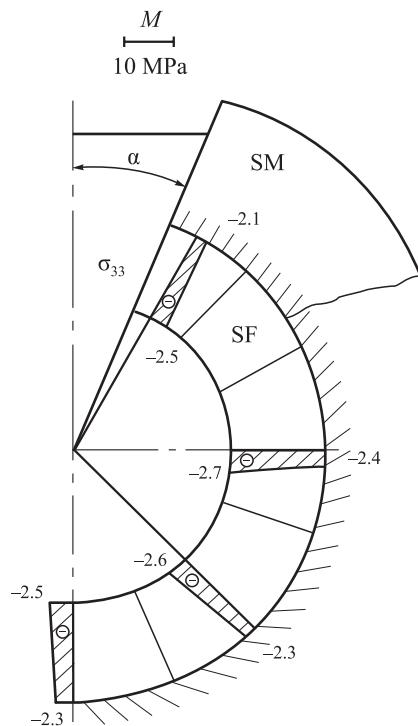
## CONCLUSIONS

A more accurate solution to the problem was obtained by considering the temperature-dependent change in



**Fig. 8.** Temperatures field (a), diagrams of normal stresses  $\sigma_{22}$  (b),  $\sigma_{33}$  (c) and  $\sigma_{11}$  (d) at  $\alpha = 30^\circ$  after casting cooling for 1.12 s (—) and 7.12 s (---)

**Рис. 8.** Поле температур (а), эпюры нормальных напряжений  $\sigma_{22}$  (б),  $\sigma_{33}$  (в) и  $\sigma_{11}$  (г) в ОФ при  $\alpha = 30^\circ$  через время охлаждения отливки 1,12 с (—) и 7,12 с (---)



**Fig. 9.** Diagrams of normal stresses  $\sigma_{33}$  in formed metal crust after casting cooling for 7.12 s ( $\alpha = 30^\circ$ )

**Рис. 9.** Эпюры нормальных напряжений  $\sigma_{33}$  в образующейся корочке металла через время охлаждения отливки 7,12 с ( $\alpha = 30^\circ$ )

the shear modulus of the mold material, which significantly impacted the results. The analysis of the SSS in a spherical SM during the pouring of a steel casting showed

that compressive stresses  $\sigma_{22}$  and  $\sigma_{33}$  at the interface between the SM and the liquid metal at the initial cooling stage are critical. Significant compressive stresses  $\sigma_{11}$  (up to 10 MPa) at the interface between the SM and the SF indicate the possibility and necessity of further theoretical study of this process by modeling the surface coverage area of the SF.

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