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Оригинальная статья

## STRESS-STRAIN STATE OF CERAMIC SHELL MOLD DURING FORMATION OF SPHERICAL STEEL CASTING IN IT. PART 1

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**Abstract.** The task of the present theoretical investigation was to determine the external factors at which a spherical shell mold will not fail due to temperature stresses occurring in it. The problem is formulated for determining the stress-strain state of the spherical shell mold formed in the support filler at cooling of solidifying spherical steel casting. The investigated axisymmetric rotational body has four zones: liquid metal, solid metal, shell mold, and support filler. To solve the problem, the equation of linear elasticity, the equation of heat capacity and a well-proven numerical method were used according to which the investigated zone is partitioned into elements by a system of orthogonal surfaces. For each element, a formulated system of equations is written in difference form, taking into account axial symmetry through the values of stresses and displacements along the element edges and the lengths of the ribs' arcs that limit its volume. The heat conduction equation is written in difference form for construction of a heat balance for an arbitrary orthogonal element, including both average temperature of the element and temperatures of the elements surrounding its volume. The authors found the solution of the difference analogue of heat equation by the "sweep" method according to the compiled iterative scheme. A difference analogue of the formulated system of differential equations of the linear theory of elasticity has the form of an algebraic system of equations. The algorithm for convolution of this system allows one to significantly reduce its rank. A general numerical scheme and algorithm for solving the problem are presented. The result of the solution is the magnitude of stresses, displacements on average along the edges of each element and average temperature in the element.

**Keywords:** investment casting, shell mold, stressed state, modeling, crack resistance

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## НАПРЯЖЕННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ КЕРАМИЧЕСКОЙ ОБОЛОЧКОВОЙ ФОРМЫ ПРИ ФОРМИРОВАНИИ В НЕЙ СТАЛЬНОЙ ШАРООБРАЗНОЙ ОТЛИВКИ. ЧАСТЬ 1

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**Аннотация.** Задачей настоящего теоретического исследования является определение внешних факторов, при которых сферическая оболочковая форма (ОФ) не будет разрушаться от возникающих в ней температурных напряжений. Сформулирована задача по определению напряженно-деформируемого состояния (НДС) ОФ, заформированной в опорный наполнитель (ОН), при охлаждении в ней затвердевающей шарообразной стальной отливки. Рассматриваемое осесимметричное тело вращения имеет четыре области (жидкий металл, твердый металл, оболочковая форма, опорный наполнитель). Для решения задачи авторы используют уравнение линейной теории упругости,

уравнение теплопроводности и апробированный численный метод, согласно которому исследуемая область разбивается системой ортогональных поверхностей на элементы. Для каждого элемента записана система уравнений в разностном виде с учетом осевой симметрии через напряжения и перемещения по граням элемента и длинам дуг ребер, ограничивающих его объем. Уравнение теплопроводности записано в разностном виде из построения теплового баланса для произвольного ортогонального элемента, включающее как среднюю температуру элемента, так и температуры элементов, окружающих его объем. Решение разностного аналога уравнения теплопроводности осуществляется методом «прогонки» по составленной итерационной схеме. Приведен разностный аналог сформулированной системы дифференциальных уравнений линейной теории упругости в виде алгебраической системы уравнений. Представленный алгоритм свертки этой системы позволяет значительно понизить ее ранг. Приводится общая численная схема и алгоритм решения задачи. Результатом решения являются величины напряжений, перемещений в среднем по граням каждого элемента и средняя температура в элементе.

**Ключевые слова:** литье по выплавляемым моделям, оболочковая форма, напряженное состояние, моделирование, трещинообразование

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## INTRODUCTION

Numerous analytical [1; 2] and theoretical studies [3 – 5] have investigated the crack resistance of a ceramic shell mold (CSM) after liquid metal is poured into it and the solidifying casting is cooled. A glass-shaped shell mold bounded by spherical and cylindrical surfaces was studied. It was established that the most dangerous stresses that arise when liquid metal is cooled in the shell mold are normal tensile stresses on the outer surface of the mold adjacent to the support filler. The researchers determined the optimal external force and temperature influences on the shell mold to guarantee its durability during steel casting production. Additionally, they studied the morphological structures of shell molds that can withstand the thermal stresses of cooled castings and proposed new designs.

Numerous theoretical and experimental studies have been conducted to establish the characteristics of the stress-strain state (SSS) of a ceramic shell mold and the resulting castings in investment casting, and the SSS dependence on factors such as the consumable pattern [6; 7], the shape and geometry of the ceramic shell mold [8; 9], mold wall thickness [10; 11], mold material [12; 13], geometry of castings [14 – 16], and methods of mold strength tests [17; 18].

Other works describe mathematical modeling of these processes, including modeling methods [19], research methods [20 – 22], studies based on numerical modeling [23 – 25], special mathematical models [26 – 28], and software [29; 30].

Further theoretical studies have shown that the durability of shell molds largely depends on their shape, which is organically related to the geometry of the casting formed in them.

However, practically no studies have investigated the modeling of the crack resistance of a ceramic shell mold based on the quantitative and qualitative indicators of its stress-strain state as a steel casting in the shape of a sphere (ball) is formed in it. This issue is addressed in our study.

We conducted a theoretical study related to steel casting production in a spherical shell mold. A variety of parts have spherical shapes, including spherical joints, which are crucial components in mechanical engineering and robotics.

The first theoretical results of the investigated manufacturing process were published in [31], where it was clearly shown that the stress-strain state in a spherical shell mold is drastically different from that in a cylindrical shell mold when producing a steel casting. However, the paper [31] does not present a mathematical model of the process.

This study shows that high-quality billets for spherical joints can be obtained using casting, which is much cheaper than metal treatment under pressure.

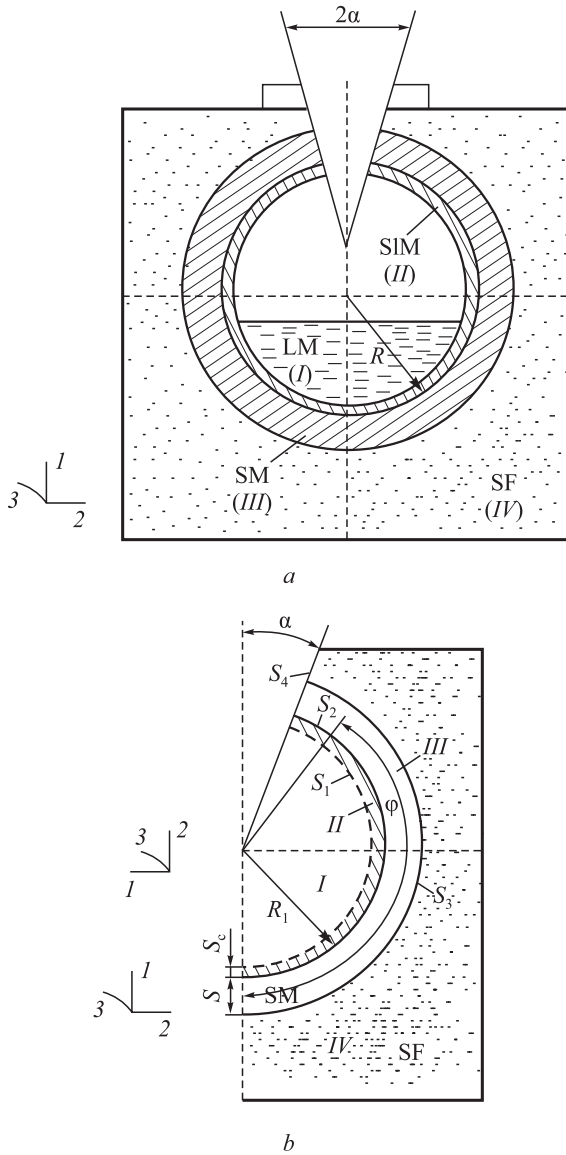
## ENGINEERING PROBLEM STATEMENT

Liquid steel is poured into a spherical mold, where it crystallizes by removing heat through the shell mold walls via the support filler (Fig. 1, *a*). A spherical shell mold can be monolithic or consist of several layers [1], each with its own physical and mechanical properties. When steel is cooled in a shell mold, temperature stresses arise in the wall due to a large temperature gradient. Under certain external influences, these stresses can lead to wall destruction, resulting in the rejection of the steel casting. Therefore, the objective of this theoretical study is to determine the external factors under which the spherical shell mold will not be destroyed by the temperature stresses arising in it.

## MATHEMATICAL PROBLEM STATEMENT

An axisymmetric rotational body is under consideration. The deformable material is considered isotropic, and the movement is deemed slow.

We have a four-component system (Fig. 1, *b*). The deformable medium includes the solidified metal (region *II*) and the mold (region *III*), both of which are isotropic materials. The process is non-stationary. Using the theory



**Fig. 1.** General (a) and calculation (b) diagrams of a spherical shell mold (SM) molded in support filler and poured with liquid metal in accordance with the axial symmetry:

LM – liquid metal (area I); SIM – solid metal (area II); SM – shell mold (area III); SF – support filler (area IV);  $S_1$  – inner contact surface of liquid and solidified metal;  $S_2$  – inner contact surface of solidified metal and shell mold;  $S_3$  – outer surface of the shell mold;  $S_4$  – free surface of the end face of casting cup;  $R$  – radius of the spherical casting;  $S$  – thickness of shell mold;  $S_c$  – thickness of solidified metal crust;  $\alpha$  – slope angle of casting cup;  $\phi$  – angle of enclosing surface of shell mold with a support filler

**Рис. 1.** Общая (a) и расчетная (b) схемы шарообразной ОФ, заформованной в опорный наполнитель и залитой жидким металлом с учетом осевой симметрии:

LM – жидкий металл (область I); SIM – твердый металл (область II); SM – оболочковая форма (область III); SF – опорный наполнитель (область IV);  $S_1$  – внутренняя поверхность контакта жидкого и затвердевшего металла;  $S_2$  – внутренняя поверхность контакта затвердевшего металла и оболочковой формы;  $S_3$  – внешняя поверхность оболочковой формы;  $S_4$  – свободная поверхность торца литниковой чаши SM;  $R$  – радиус шарообразной отливки;  $S$  – толщина оболочковой формы;  $S_c$  – толщина корочки затвердевшего металла;  $\alpha$  – угол наклона литниковой воронки;  $\phi$  – угол охвата поверхности оболочковой формы опорным наполнителем

of elasticity and the Eulerian reference frame, we write a system of equations for each region:

– for region I:

$$\begin{aligned}\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma = P; \\ P = \gamma h; \theta = \alpha_1 \Delta \theta;\end{aligned}\quad (1)$$

– for regions II, III:

$$\begin{cases} \sigma_{ij} = 0, i, j = 1, 2, 3; \\ \sigma_{ij} - \sigma \delta_{ij} = 2G_p \varepsilon_{ij}^*; \varepsilon_{ij}^* = \varepsilon_{ij} - \frac{1}{3} \varepsilon \delta_{ij}; \varepsilon = \varepsilon_{ii}; \\ \varepsilon_{ii} = 3k_p \sigma + 3\alpha_p (\theta - \theta_p^*); \varepsilon_{ij} = 0, 5(U_{ij} + U_{ji}); \\ C_p \gamma \frac{\partial \theta}{\partial \tau} = \text{div}(\lambda \text{grad} \theta); \end{cases}\quad (2)$$

where  $\sigma_{ij}$  are the components of the stress tensor;  $\sigma$  is the hydrostatic stress;  $\varepsilon_{ij}$  are the components of the elastic strain tensor;  $h$  is the height of the liquid metal column;  $k_p = \frac{1-2\mu}{E}$  is the coefficient of volume compressibility;

$\mu$  is Poisson's ratio;  $E$  is Young's modulus;  $G_p$  is the shear modulus in region  $p$  (II, III);  $\alpha_p$  is the linear expansion coefficient;  $\alpha_1$  is the temperature conductivity coefficient in the region I;  $\tau$  is time;  $\theta$  is temperature;  $C_p$  is the specific heat in the region  $p$ ;  $\gamma$  is density;  $\theta_p^*$  is the initial temperature in the region  $p$ ;  $\lambda = \lambda(\theta)$  is heat conduction;

$\sigma_{ij,j} = \frac{\partial \sigma_{ij}}{\partial x_j}$ ;  $u_{i,j} = \frac{\partial u_i}{\partial x_j}$ ; and summation over repeated indices is performed.

In accordance with axial symmetry, we consider the meridian section (Fig. 1, b).

If  $\theta_m \leq \theta_{cr}$  ( $\theta_m$  and  $\theta_{cr}$  are the temperatures of the metal and crystallization), as the liquid metal is cooled, its temperature is determined by the thickness of the solidified layer  $\Delta_i$  from the solution of the interphase transition equation [5].

Initial statement of the problem:

$\Delta|_{\tau=0} = 0$  (absence of the metal solid phase);

$\theta|_{\tau=0} = 0 = \theta_m^*$  (temperature of the poured liquid metal);

$\theta_{III}|_{\tau=0} = \theta^*$  (initial mold temperature).

Boundary conditions of the problem (Fig. 1, b):

– on the axis of symmetry:  $U_2 = 0$ ;  $\sigma_{21} = 0$ ;  $q_n = 0$ ;

– on surfaces  $S_1, S_3, S_4$

$$\begin{aligned}\sigma_{11}|_{S_1} = -P; \sigma_{12}|_{S_1} = 0; U_1|_{S_3} = 0; \sigma_{22}|_{S_4} = 0; \\ \sigma_{12}|_{S_3} = -\psi \frac{U_{ck}}{U^*} \text{con}(n_1 x_1); \theta|_{S_3} = \theta^*;\end{aligned}\quad (3)$$

where  $U_{ck}$  is the sliding of the mold material relative to the sand;  $U^*$  is the normalizing displacement; and  $\psi$  is a parameter characterizing the friction conditions between the mold and the support filler.

The numerical method was used [32] to solve the system (2). According to this method, the deformation region is divided into a finite number of orthogonal curved elements (Fig. 2, *a*).

With axial symmetry, we have  $\sigma_{31} = \sigma_{32} = 0$ ;  $\sigma_{13} = \sigma_{23} = 0$ ;  $U_3 = 0$ .

In accordance with [32], the equations (2) and the values  $\varepsilon_{ij}$  taking into account axial symmetry, will be written as follows:

$$\begin{aligned} S_{13}\Delta S_{12}(\sigma_{11} - \sigma_{22}) + S_{12}S_{13}(\sigma_{11} - \sigma_{33}) + \\ + 0,5\Delta\sigma_{11}S_{12}S_{13} + 0,5\Delta\sigma_{12}S_{21}S_{23} + \\ + (S_{21}\Delta S_{23} + 2S_{23}\Delta S_{21})\sigma_{21} = 0; \\ S_{21}\Delta S_{23}(\sigma_{22} - \sigma_{33}) + S_{23}S_{21}(\sigma_{22} - \sigma_{11}) + \\ + 0,5\Delta\sigma_{22}S_{23}S_{21} + 0,5\Delta\sigma_{21}S_{12}S_{13} + \\ + (S_{12}\Delta S_{13} + 2S_{13}\Delta S_{12})\sigma_{21} = 0; \end{aligned} \quad (4)$$

$$\begin{aligned} \sigma_{11} - \sigma_{22} = 2G_p(\varepsilon_{11} - \varepsilon_{22}); \\ \sigma_{22} - \sigma_{33} = 2G_p(\varepsilon_{22} - \varepsilon_{33}); \end{aligned} \quad (5)$$

$$\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = 3k_p\sigma + 3\alpha_p(\theta - \theta_p^*); \quad (6)$$

$$\begin{aligned} \varepsilon_{11} = \frac{2\Delta U_1}{S_{21}} + \frac{2U_2}{S_{21}} \frac{\Delta S_{21}}{S_{12}}; \quad \varepsilon_{22} = \frac{2\Delta U_2}{S_{32}} + \frac{2U_1}{S_{12}} \frac{\Delta S_{12}}{S_{21}}; \\ \varepsilon_{33} = \frac{2U_1}{S_{13}} \frac{\Delta S_{13}}{S_{31}} + \frac{2U_2}{S_{23}} \frac{\Delta S_{23}}{S_{32}}; \end{aligned} \quad (7)$$

where  $U_i = U_i^1 + U_i^2$ ,  $\Delta U_i = U_i^2 - U_i^1$ , ( $i = 1, 2$ );  $S_{ij} = S_{ij}^1 + S_{ij}^2$ ;  $\Delta S_{ij} = S_{ij}^2 - S_{ij}^1$ .

The adopted symbols are described in [1; 7].

The equations (4) – (7) are written taking into account

that  $\frac{\partial U_1}{\partial x_3} = 0$ ,  $\frac{\partial \sigma_{3i}}{\partial x_3} = 0$ ;  $i = 1, 2, 3$ ; for rotational bodies

$\Delta S_{31} = 0$ ;  $\Delta S_{32} = 0$ ;  $\frac{\Delta U_1}{S_3} = 0$ ;  $\frac{\Delta U_2}{S_3} = 0$ ;  $U_3 = 0$ ; on the sur-

face  $x_1x_3$ :  $S_2^+ - S_2^- = 0$ ; on the surface  $x_2x_3$ :  $S_2^+ - S_2^- = 0$ ; shift values  $\varepsilon_{ij}$  ( $i \neq j$ ) shall be written for the node (0) (Fig. 2, *d*) in the form

$$\begin{aligned} \varepsilon_{12}^0 = \frac{2\Delta \bar{U}_2}{S_1} - 0,5\bar{U}_2 \frac{S_2^+ - S_2^-}{S_1S_2} + \\ + \frac{2\Delta \bar{U}_1}{S_2} - 0,5\bar{U}_1 \frac{S_1^+ - S_1^-}{S_1S_2}, \end{aligned} \quad (8)$$

where  $S_i = S_i^1 + S_i^2$ ;  $\Delta \bar{U}_i = \bar{U}_{i2} - \bar{U}_{i1}$ ;  $S_i^+ = S_i^{1+} + S_i^{2+}$ ;  $S_i^- = S_i^{1-} + S_i^{2-}$ ;  $\bar{U}_i$  is the average of the value  $U_i$  along the element edges.

The authors of [32] prove that if there are initial and boundary conditions, the difference analogue of the system (4) – (6), taking into account the equation (7), is

definable. The order of the system (4) – (6) is significantly reduced when the following operations are performed.

1. The differences  $(\sigma_{ij} - \sigma_{jj})$  in equations (4) are expressed by formula (5).

2. The mass conservation equation is rewritten in the recurrent form, taking into account expressions (7), in the form  $U_1^2 = U_1^1 + [A]$ ; where  $[A]$  is an operator that does not contain  $U_1^2$ ; direction of bypassing the area along  $x_1(\rightarrow)$ , along  $x_2(\uparrow)$ .

3. Shift expressions  $\varepsilon_{ij}$  ( $i \neq j$ ) are determined based on internal grid nodes in accordance with formulas (8);  $i = 1, j = 2$ .

4. The values  $\sigma_{ij}$  ( $i \neq j$ ) are determined based on the internal grid nodes from the equations of state  $\sigma_{12}^0 = G_p^0 \varepsilon_{12}^0$ .

5. The values  $\sigma_{ij}$  are determined based on the external grid nodes from the boundary conditions, and on the contact surfaces – from the friction law.

6.  $\sigma_{ij}$  are determined based on the element edges as the average of the values  $\sigma_{ij}$  at the edges' nodes.

7. The first equation (4) is rewritten in the recurrent form  $\sigma_{11}^1 = \sigma_{11}^2 + [E]$ ; where  $[E]$  is an operator that does not contain  $\sigma_{11}^1$ ; direction of bypassing the area along  $x_1(\leftarrow)$ , along  $x_2(\downarrow)$ .

8. From the system of equations (the second equation in the system (4) and the equation  $\sigma_{22} - \sigma_{11} = 2\lambda(\varepsilon_{11} - \varepsilon_{22})$ ) the values  $\sigma_{22}^1$  and  $\sigma_{22}^2$  for the element are determined, the equations of the form  $F_3 = (\sigma_{22}^2)_J - (\sigma_{22}^1)_{J+1} = 0$  composed for the internal edges (where  $J$  is the element index on the coordinate  $x_2$ ).

Thus, if we consider  $X = \{U_2, U_1|_{x_1=0}, \sigma_{11}|_{x_1=x_1^*}\}$ , independent variables, the sequence (1) – (7) can be used to determine the dependent variables through  $X$  ( $x_1^*$  is the final value of the coordinate  $x$  in the curved region).

The equivalent system of equations looks as follows

$$\begin{aligned} F_1 = (U_1^2 - U_1^*)|_{x_1=x_1^*} = 0; \\ F_2 = (\sigma_{11}^1 - \sigma_{11}^*)|_{x_1=0} = 0; \\ F_3 = 0; \end{aligned} \quad (9)$$

where  $U_1^*$  is known from the boundary conditions of displacement  $U_1$  at the boundary of the region ( $x_1 = x_1^*$ );  $\sigma_{11}^*$  is known from the boundary conditions of stress  $\sigma_{11}$  at the boundary of the region ( $x_1 = 0$ ).

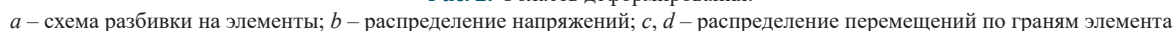
There are as many equations  $F_1 = 0$  as there are unknown variables  $\sigma_{11}|_{x_1=x_1^*}$ , and as many equations  $F_2 = 0$  as there are unknown variables  $U_1|_{x_1=0}$ .

The coefficients and free terms of the new equivalent system of equations (9) can be found using the following procedure.

Suppose the equivalent system of equations looks as follows

$$\bar{F}_i = \alpha_{ij}x_j + b_i = 0; \quad i, j = 1, \dots, n. \quad (10)$$

where  $\theta_k^*$  is the average temperature in the  $k^{\text{th}}$  element at the beginning of the time step  $\Delta\tau$ ;  $\lambda_k$ ,  $\theta_k$ ,  $C_k$ ,  $\gamma_k$  are the average heat conduction, temperature, heat capacity, and density in the  $k^{\text{th}}$  element at the end of the time step  $\Delta\tau$ ;  $\lambda_i^-$ ,  $\theta_i^-$  and  $\lambda_i^+$ ,  $\theta_i^+$ , ( $i = 1, 2$ ) are the heat conduction and tem-



perature in the element following the element  $k$  on the coordinate  $x_i$  in the negative and positive directions  $x_j$ ;  $S_{21}^- = S_{21}^{1-} + S_{21}^{2-}$ ;  $S_{21}^+ = S_{21}^{1+} + S_{21}^{2+}$ ;  $S_{ij}^{1+}$  ( $i \neq j$ ;  $i, j = 1, 2$ ) and  $S_{ij}^{1-}$  are the length of the arc  $S_{ij}^1$  of the element following the element  $k$  in the positive and negative directions on the coordinate  $x_j$ .

The authors of [32] proved the iteration procedure convergence (11).

#### ALGORITHM FOR SOLVING THE PROBLEM

1. The cooling time  $\tau^*$  is divided into a finite number of steps:  $\tau^* = \sum \Delta\tau_n$ ; where  $n$  is the number of the time step.

2. The region under study is divided into a finite number of orthogonal elements.

3. The initial and boundary conditions for the elements forming the investigated region and the constants of the physical and mechanical properties of the materials are established.

4. The lengths of the elements' arcs are calculated  $S_{ik}^j$  ( $i, k = 1, 2$ ;  $i \neq k$ ;  $j = 1, 2$ ).

5. The temperature field at the time step  $\Delta\tau_n$  is determined by the numerical solution of the heat equation using the iterative formula (11) if there are initial and boundary conditions at the investigated time step.

6. If the temperature in the region  $I$  near the surface  $S_2$   $\theta|_{S_2} \leq \theta_k$ , the thickness  $\Delta_n$  of the crystallized crust is calculated [5]. If  $\theta|_{S_2} > \theta_k$ , the following operation is performed.

7. The system of equations (2) is solved taking into account difference analogues (4) – (7) and the developed procedures [1; 32] described above. The fields of stress  $\sigma_{ij}$  and displacements  $U_i$  ( $i, j = 1, 2$ ) are determined.

8. Time steps are taken; if  $\sum \Delta\tau_n < \tau^*$ , the operation 4 is performed; if  $\sum \Delta\tau_n = \tau^*$  the calculation process is completed.

As the temperature problem was solved, boundary conditions of the first kind (3) were used. To determine  $\theta_m(\tau)$  and  $\theta^*(\tau)$ , we will use the experimental data from [1]. Approximation of these values yields the following result:

$$\theta_m = 1550 - 1.666\tau - \frac{\tau(60 - \tau)}{10 + \tau^2};$$

$$\theta \leq \tau \leq 60 \text{ s};$$

$$\theta^* = 20 + 17.3\sqrt{\tau};$$

where  $\tau$  is the cooling time, s.

The time  $\tau$  does not exceed 60 s, since at  $\tau \geq 60$  s the stresses in the shell mold plummet and cannot cause its destruction.

A mathematical model has been developed to determine the stress-strain state and temperature in the shell mold when a spherical casting is cooled in it. This model was used in a numerical solution of the problem on modeling the crack resistance of a spherical shell mold.

#### CONCLUSIONS

The first theoretical attempt was made to formulate and solve the problem of determining external factors at which a spherical shell mold will not fail due to temperature stresses occurring in it.

Based on the fundamental equations of the theory of elasticity and numerical methods, a numerical scheme and algorithm for solving the problem were developed.

The proposed method for modeling the crack resistance of a spherical shell can be recommended for other functional shells.

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